The Königsberg bridges
The old town of Königsberg has seven bridges. The famous mathematician Euler asked himself: "can you take a wall through the town, visiting each part of the town $(A, B, C$ and $D)$, and crossing each bridge only once?

A


Let's simplify the problem
Each dot is a zone in the city. Connect the zones/dots using lines as the bridges Now Euler's question is: can you draw each line only once without lifting your pencil?
$O^{A}$

Paths on shapes
Draw these shapes without lifting your pencil and without retracing any line


A


B


C


D

| Shape | A | B | C | D |
| :--- | :--- | :--- | :--- | :--- |
| Success? |  |  |  |  |

Arriving and leaving the dots.
While drawing the shapes, how many times can you arrive and leave the dot using each line only once?


A


B


C


D
D

| Shape | A | B | C | D |
| :--- | :--- | :--- | :--- | :--- |
| Number of lines | 4 | 3 |  |  |
| How many times you <br> crossed the dot? | 2 |  |  |  |
| Any line left? | No! |  |  |  |

Do you see a pattern? Which ones can be only at the beginning or the end of our path?

Pattern:

- Odd number of lines: those dots can only be the beginning or the end of our path.
- Even number of lines: those dots can only be in the middle of our path.

So, lef's try again


A


B


C


D

| Shape | A | B | C | D |
| :--- | :--- | :--- | :--- | :--- |
| Number of dots with even number of lines | 4 |  |  |  |
| Number of dots with odd number of lines | 0 |  |  |  |
| Success? | Yes! |  |  |  |

What about the Krönisgberg problem? How many dots with even number of lines? Odd number? Can Euler walle the bridges as he proposed?


